

Inductance of Solenoids on Ferrite Rods

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Something new is offered here for predicting the self inductance of loop antenna coils wound on ferrite rods. While working up an article on design of loop antennas, we struggled in vain to find formulas or algorithms for accurate prediction of inductance. Finally, through a lot of simulation work, some discoveries led to this proposal of a new method for predicting inductance.

This article takes a decidedly empirical approach to the problem. Rather than present equations based on field theory, inductance estimates based on the analysis of data from electromagnetic simulations are proposed. These have been verified against published data and local experiments and found to be more accurate than other methods we are aware of. A zip file is attached to this PDF document containing sample Matlab/Octave scripts and data which implement the methods proposed herein.

The new inductance estimate uses the familiar inductance factor:

$$L = A_L n^2 ,$$

but with a new definition for that factor:

$$A_L = \mu_o \mu_L \sqrt{l_f d_f}$$

This formula includes the ferrite rod's length-diameter product and a new relative permeability term, μ_L , called inductive permeability. It's is a function of coil length, intrinsic permeability of the ferrite and the rod's aspect ratio (length over diameter). Inductive permeability values are based on results from hundreds of electromagnetic simulations.

Data tables and interpolation functions are attached to the PDF file containing this document for the purpose of computing inductive permeability. A table of values for μ_L is also included at the end of this article.

We begin with a quick review of methods currently used to predict inductance. MKS units are assumed in this article unless otherwise noted.

Internal Permeability – μ_{int}

There are two common approaches to estimating inductance of a ferrite-cored antenna solenoid. The ratio between coil inductance with and without the ferrite core is one approach, and it defines a relative permeability value. This is known by a couple of different names,

$$\mu_{int}, \mu_{coil} \equiv \frac{L_{ferrite}}{L_{air}}$$

We found this measure of permeability to be of limited value. What makes it particularly unattractive is that coil and wire diameter have a big effect on air core inductance, but within reason, these parameters have little to do with the inductance when a ferrite rod core is present. By changing coil and/or wire diameters, a range of values for μ_{int} may be measured and this really has nothing to do with the characteristics of the ferrite rod.

Figure 1 shows published measurements and calculations of μ_{int} (see [Bolton-1]), and demonstrates the uncertainty in this approach. At the very least, for μ_{int} to be useful there would need to be some standards applied to the coil geometry. Because of these issues, no further attention is given to μ_{int} here.

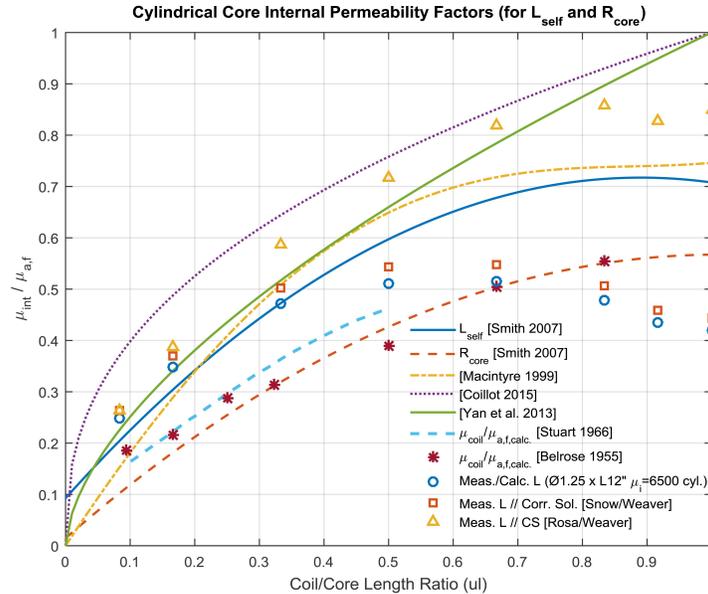


Figure 1: μ_{int} From Bolton

Inductance Factor

In this more promising approach, the self inductance of an n-turn coil wound on a ferrite core is estimated by this very simple formula:

$$L = A_L n^2 \tag{1}$$

where A_L is the so-called *inductance factor*. In the MKS system, it has units of Henries per turn-squared. It encapsulates several issues related to the intrinsic permeability of the ferrite material (μ_i), the geometry of the ferrite rod and coil length (relative to the rod length) into a single factor. This equation to estimate the inductance factor is found in many places, including section 4.3.4 of [Snelling]:

$$A_L = \mu_o \mu_{rod} \frac{A_f}{l_f}$$

A_f is the cross sectional area of the rod, and l_f its length. The term μ_{rod} is sometimes referred to as the fluxmetric relative permeability. This estimate for A_L is advertised to be applicable only to full-length coils ($l_c = l_f$), but even then it doesn't seem to work all that well. Errors of 15% are not uncommon. Larger adjustments are required for coils shorter than the rod length. We believe this is better than using the μ_{int} parameter because it does not depend on the coil or wire diameter. Still, it would be useful to have a more accurate estimate of inductance, one that takes coil length into account. What is needed is a more accurate estimate of A_L .

A Fix for A_L

Electromagnetic simulations were performed where rod parameters (diameter, length and permeability) and coil length were held constant, but coil turn count and wire diameter were varied. It was found that inductance in this case could be accurately predicted from the turn count alone, and it is indeed proportional to the square of turn count as suggested in formula (1). Experiments with physical rods and coils confirmed this as well. For this relationship to hold, coil length must be held constant, so coils with more turns will have a smaller winding pitch (turn spacing).

With this in mind, a large number of simulations were run covering a three-dimensional grid of values for μ_i , ferrite rod aspect ratio (length over diameter, l_f/d_f), and coil-to-rod length ratio (l_c/l_f). In all cases the coil was centered on the rod. Based on the results, a new equation for inductance factor is proposed. Regrettably, it requires the introduction of a new relative permeability term. While it would be possible to redefine an existing term, it was felt this would be even worse than defining a new term.

Starting with a common formula for computing inductance factor,

$$A_L = \mu_o \mu_{rod} \frac{A_f}{l_f} = \mu_o \mu_{rod} \frac{\pi}{4} \frac{d_f^2}{l_f}$$

where d_f is the rod diameter, we propose replacing μ_{rod} with the following:

$$\mu_{rod} \Leftarrow \mu_L \frac{4}{\pi} \left(\frac{l_f}{d_f} \right)^{3/2} \quad (2)$$

It is *not* claimed this replacement is a good estimate for μ_{rod} . Rather, it's an assertion that μ_{rod} doesn't belong in this formula and that the proposed replacement is a better fit.

While it would have been possible to leave μ_{rod} in there and insert another correction function, replacing it as shown produces a simpler result. More detail on the reasoning behind this decision is provided later in the article.

The substitution leads to this result:

$$A_L = \mu_o \mu_L \sqrt{l_f d_f} \quad (3)$$

It may seem like a strange choice, but this was suggested by an analysis of dozens of simulations.¹ The extra $4/\pi$ term in (2) is arbitrary, and included only to make the result in (3) simpler – we could have omitted the $4/\pi$ term and rolled it into μ_L instead.

- The term $\sqrt{l_f d_f}$ came as a surprise, but it works. It takes care of a majority of the dependence on ferrite rod geometry.
- μ_L is the new relative permeability term. It is a function of coil length, rod aspect ratio and the ferrite material's intrinsic permeability.
 - The ratio of coil to rod length (l_c/l_f) has a major effect, accounting for most of the variation in μ_L .
 - Intrinsic permeability μ_i and rod aspect ratio (l_f/d_f) play a less significant but important role.

This new proposed formula for inductance factor A_L can give estimates that are accurate to 2-3% in many cases, and significantly better than with previous formulas.

¹Actually, the dependence on square root of length-diameter product was discovered first, then working backwards led to the substitution shown here.

Inductive Permeability – μ_L

It seems fitting to call this new relative permeability term *inductive permeability*. It’s suspiciously like the “constant of proportionality” described in section 4.3.4 of [Snelling], which is graphed in his figure 4-12 and reproduced here as figure 2. We would have used Snelling’s symbol for this term here but he did not define a symbol to represent it.

Snelling must have done a lot of experimental work to produce this graph. Although useful, it only predicts a range of inductance, rather than specific values. A lot more experimental work would have been required to extract the dependencies that were revealed in our simulations, so it’s not surprising this sort of characterization hasn’t been done before.

Figure 3 shows the μ_L function for an intrinsic permeability of 500. It is remarkably similar to Snelling’s graph of the “constant of proportionality”. Snelling provides a shaded band within which the correct value lies, while the μ_L function provides a specific answer for each combination of μ_i and aspect ratio. Not only do the two sets of data overlap, but the simulations ran to develop μ_L reveal that the data spread in Snelling’s plot for any given coil length is due to different rod aspect ratios and values of μ_i .

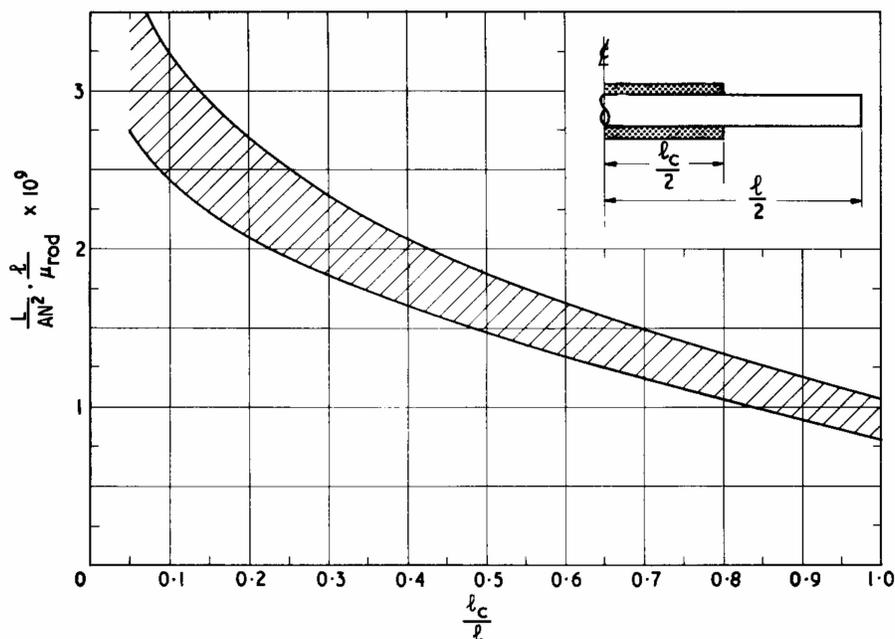


Fig. 4.12. $L.l/AN^2\mu_{rod}$ as a function of l_c/l . The results for a wide variety of ferrite rods are contained within the shaded area, (dimensions in mm, L in H)

Figure 2: Constant of proportionality reproduced from [Snelling]

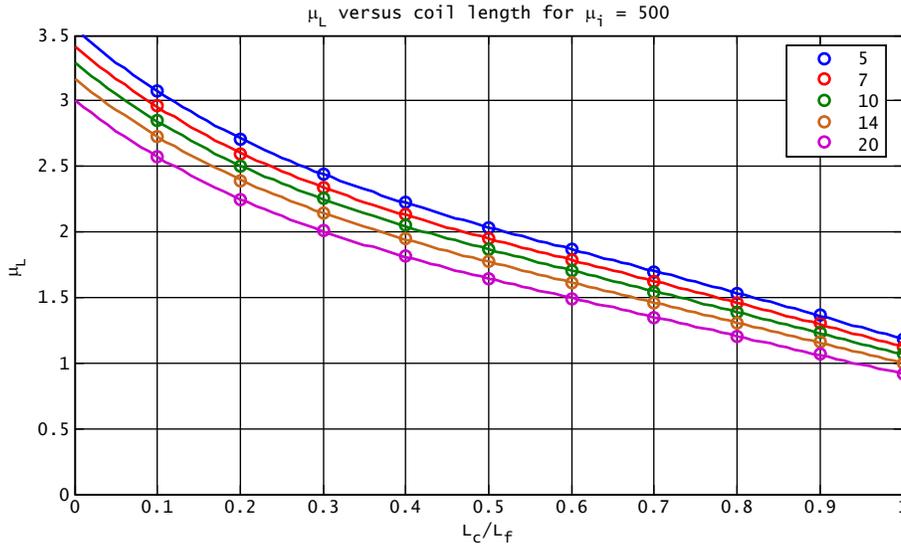


Figure 3: Inductive permeability μ_L for $\mu_i = 500$ and l_f/d_f from 5 to 20

Estimating μ_L

Simulation data was generated in 350 runs over a uniform three dimensional grid of μ_i , rod aspect ratio and coil-to-rod length ratio:

- Seven values of μ_i from 63 to 4000
- Five rod aspect ratios from 5 to 20
- Ten coil-to-rod length ratios from 10 to 100 percent of the rod length

This covers most of the practical geometries and ferrite materials currently in use. In most cases this data must be interpolated to a given point of interest – specific values of μ_i , rod aspect and coil-to-rod length ratios.

It may be possible to extrapolate to rod aspect ratios as small as 3 and as large as 30, but no further than that. For rod aspect ratios of 20 or less, estimating inductance for permeability values above 4000 may can be done using the data for $\mu_i = 4000$.

For points of interest which lie wholly within the range of tabulated values we recommend using a three dimensional spline interpolation. For points lying outside the range of tabulated values, polynomials may be fit to all data points along the dimension in question to provide a reasonable extrapolation for a short distance. This reduces the number of dimensions, and the remaining data can then be interpolated using splines.

A zip file attached to the PDF file containing this article contains μ_L values from simulations along with a sample Matlab/Octave script which performs the interpolation.

Verification of Inductance Estimates

Comparisons of interpolated simulation data to published data and locally performed experiments has been done in an attempt to verify these predictions. Those results are presented here.

Bolton's Examples

Twelve sample ferrite core antennas measured by [Bolton] are compared to predictions of inductance as proposed. Figure 4 shows this comparison. Those rods are said to have $\mu_i = 6500$ which is quite a bit larger than the largest simulation data point ($\mu_i = 4000$), although with such high values of μ_i and the largest rod aspect ratio less than ten, we wouldn't expect the shift from 4000 to 6500 to have a huge impact.

The two data points that are about 8% low are discouraging, but still not all that far off. Without knowing more about Bolton's work and/or having access to his samples, not much more can be said about the discrepancies.

ID	Predicted	Measured	Percent Diff
1	76.3	69.8	-8.6
2	1310.2	1331.1	1.6
3	65.8	63.9	-2.8
4	1118.9	1154.5	3.2
7	63.1	57.8	-8.3
12	66.9	65.3	-2.4
13	75.8	73.0	-3.7
14	89.3	86.5	-3.2
15	101.9	101.3	-0.5
16	120.2	118.1	-1.8
17	141.4	138.6	-2.0
18	157.2	155.4	-1.1

Figure 4: Predicted inductance versus Bolton's Measurements

Local Experiments

Several ferrite rods were available for experimentation, but only two of them were of a known material, and those two have a 20% tolerance on intrinsic permeability. Therefore, it was not possible to completely verify the new inductance formulas. Instead, attempts were made to estimate the value of μ_i for each rod. If the methods developed here are accurate, it should be possible to find a reasonable value of μ_i for each rod, for which all of the inductance estimates closely match the measured values.

A total of eight different ferrite rods were paired with a variety of test coils. Some of these are pictured in figure 5. In each case, the coil was approximately centered, then adjusted slightly for maximum inductance. Figures 7

and 8 contain the results. Values for μ_i were adjusted manually to get the predictions to match measurements as well as possible.



Figure 5: Some of the rods and coils tested

Inductance measurements were made at a frequency of 10kHz for all but the one turn coils, which were measured at 100kHz. This is well below the self-resonance frequency (SRF) for any of the coil/rod combinations, and corrections for SRF in the inductance values were not made.

A one-turn coil was included to cover the widest possible range of relative geometries. In general, coils were no more than 16mm in diameter. However, coils with 21, 31 and 45 turns were 25-27mm in diameter. Data for coils significantly larger in diameter than the rods is included to illustrate two points:

- Within reason, the diameter of a coil on a ferrite rod has little effect on its self inductance, and our proposed formula still applies.
- The μ_{int} parameter is not all that useful for characterizing ferrite rods.

Regarding the second point above, air-core inductance of the 21-turn coil was about $3.5\mu\text{H}$ which gives tiny values for μ_{int} in the neighborhood of 10 or so – much smaller than μ_{rod} . If a smaller diameter coil with the same length is used to compute μ_{int} , a much larger value results. If μ_{int} is to provide useful information about the ferrite rod, it should not be so heavily dependent on coil diameter.

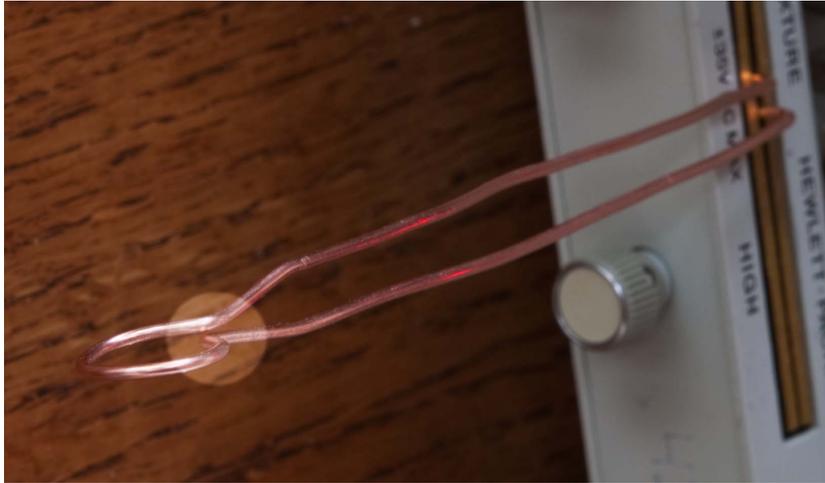


Figure 6: One-turn coil

The One-Turn Coil

Some care was required in setting up this test. Since the inductance varies as turn count squared, a small error in the actual radial angle encompassed by the coil can have a significant effect when the turn count is only one. The turn count may be expressed in degrees, with 360 degrees being exactly one turn, 540 being $1\frac{1}{2}$ turns and so on. Increasing the turn count by one percent – from 360 to 363.5 degrees would increase the inductance by two percent. For a coil with 8mm radius, this amounts to an increase of only 0.5mm in wire length making up the coil.

For the measurements to be accurate, the coil and ferrite rod must be some distance from metallic objects such as the chassis of the LCR meter used to make measurements. Thus, a significant amount of lead length is required and it must be removed from the measurement. That's not such a big deal with most of the coils, but it matters a lot with a single turn.

The coil is shown in figure 6, and it keeps the ferrite rod about 100mm away from the test fixture and LCR meter chassis. It is fabricated from 14-gauge bare copper wire for mechanical stability. The highlighted circular area in the photo shows that the two coil ends are very close to each other. This permits shorting out the coil by squeezing the ends together, which is done while zeroing the LCR meter. This can be done without significantly altering the feed wire geometry.

Simulations only considered coil-to-rod length ratios of 0.1 and above. The one turn coil requires extrapolating polynomial fits outside the range of simulation data, and this shows that the results are valid for very short coils.

Rod		Coil		Inductance μH		Percent
Dimensions	μ_i	Turns	Length	Measured	Predicted	Difference
12x200	230	1	2.0	0.165	0.160	-2.9
		21	81.0	40.0	41.1	2.9
		31	30.0	126	122	-3.2
		40	53.0	172	174	1.4
		45	40.0	234	239	1.8
		108	50.0	1309	1290	-1.5
		132	195.0	872	857	-1.8
		204	83.0	3870	3842	-0.7
12.7x190/33	625	1	2.0	0.202	0.195	-3.4
		21	81.0	51.2	52.8	3.0
		31	30.0	154	152	-1.7
		45	40.0	290	300	3.4
12.7x190/77	1200	1	2.0	0.208	0.204	-1.7
		21	81.0	55.1	56.4	2.4
		31	30.0	164	161	-2.2
		45	40.0	312	319	2.1
		157	165.0	2048	2006	-2.1
8x60	270	1	2.0	0.079	0.082	4.2
		31	30.0	48.3	46.2	-4.3
		40	53.0	49.5	51.5	4.1
		45	40.0	82.3	83.0	0.9
		96	28.5	452	454	0.4
		108	50.0	409	400	-2.3
10x80	220	1	2.0	0.103	0.106	2.5
		31	30.0	68.0	66.0	-2.8
		40	53.0	81.5	83.8	2.8
		45	40.0	122	124	1.7
		96	28.5	659	645	-2.1
		108	50.0	648	633	-2.2

Figure 7: Local inductance experiments

Rod		Coil		Inductance μH		Percent
Dimensions	μ_i	Turns	Length	Measured	Predicted	Difference
10x100	169	1	2.0	0.108	0.110	2.0
		31	30.0	73.7	71.4	-3.1
		40	53.0	91.6	94.4	3.1
		45	40.0	133	136	2.3
		96	28.5	703	695	-1.0
		108	50.0	718	709	-1.3
		204	83.0	1784	1799	0.9
		10x140 Old	167	1	2.0	0.119
		21	81.0	24.9	25.8	3.6
		31	30.0	86.2	83.3	-3.3
		40	53.0	111	115	3.6
		45	40.0	157	161	2.6
		96	28.5	815	811	-0.5
		108	50.0	857	857	-0.0
		204	83.0	2365	2400	1.5
10x140 Eton	271	1	2.0	0.128	0.129	0.8
		21	81.0	27.9	28.9	3.3
		31	30.0	94.8	91.7	-3.3
		40	53.0	125	128	2.2
		45	40.0	175	178	1.8
		96	28.5	899	891	-0.9
		108	50.0	966	952	-1.4
		204	83.0	2694	2685	-0.3

Figure 8: Local inductance experiments (continued)

Validation Summary

This is only a partial confirmation of the inductance estimation methods proposed here. The values of μ_i displayed in figures 7 and 8 are only estimates. We can provide no independent confirmation of these values. Experiments beyond our capabilities would be required to confirm the accuracy of our fits versus permeability.

For a more complete verification, μ_i values would need to be accurately measured for each rod. That would require physically slicing a small segment from each rod and boring a hole in the middle of each one so a toroidal measurement could be made. Not only don't we have the equipment to do that, but doing so would alter the permeability of each sample. It would then be necessary to anneal the toroid sample and the rod as well before making any measurements.

On the other hand, dependence of A_L on the coil-to-rod length ratio contained within the simulation data agrees with measurements typically within 2-4% and 5% worst case.

Note about μ_i

Intrinsic permeability of ferrite is altered by shock and stress, and all but two of the rods used have an unknown history of handling and it is therefore not possible to surmise what their original μ_i values may have been (even if our estimates of their current μ_i values are correct).

The estimated μ_i value for the 77-material rod is quite low (1200 versus a specification of 2000). It is a bit outside the bottom end of the specified tolerance (adjusted for temperature of the test sample). We don't know if this is correct or not. With high permeability materials, the sensitivity of A_L to changes in permeability is quite low so trying to estimate it in this way may be of questionable accuracy.

Fluxmetric Permeability – μ_{rod}

This term was introduced earlier and dismissed as not being useful without much explanation. An in depth discussion of it, along with formulas for estimation can be found in [Bolton]. Here we present our reasoning for this choice.

Perhaps the reason that this measure of permeability doesn't work as well for predicting the inductance factor has to do with it's purpose. It is measured with an externally applied *uniform* magnetic field. This is the case when the ferrite rod is exposed to a propagating electromagnetic plane wave.

Using μ_{rod} may not make so much sense when the applied field is not uniform. That is the case when the field is generated by a coil wound around

the rod.

The question of whether to use μ_{rod} is further confused by the existence several different estimates for this value. This is also discussed in our article on loop antennas [osenr], and we eventually ended up generating our own estimate of μ_{rod} based on simulation data. To illustrate this point, two different estimates for this term have been examined as described below. The first version computes μ_{rod} using a demagnetization factor proposed by Cross and published in [Bolton]. The second estimate is the one we developed as published in [osenr].

Why not just create a correction for μ_{rod} ?

The usual equation for inductance factor

$$A_L = \mu_o \mu_{rod} \frac{A_f}{l_f}$$

may be solved for μ_{rod} :

$$\mu_{rod} = \frac{A_L}{\mu_o} \frac{l_f}{A_f}$$

From this, values of A_L produced by the methods proposed here may be back-converted into values of μ_{rod} . This is equivalent to asking what value of μ_{rod} would give the same inductance factor as the new proposed methods.

By computing a large set of values for A_L using the new formulas and data, then back-converting them to μ_{rod} , it's possible to compare μ_{rod} values with what μ_{rod} would need to be to produce the same inductance estimate as our new method.

The results are shown in figure 9 for two different coil-to-rod length ratios representing long and short coils. The top two graphs are made using the μ_{rod} estimates from Cross' demagnetization factor. Bottom graphs use μ_{rod} estimates based on our simulation data as published in [osenr].

What we find depends strongly on which estimate is used for μ_{rod} . We found μ_{rod} estimates using Cross' demagnetization factor to be inaccurate for use in computing effective heights of loop antennas. However, they produce better estimates of inductance than do estimates of μ_{rod} we published in [osenr], especially for short coil lengths.

In all cases however, μ_{rod} estimates must be offset by a multiplier to line up with the back-converted data, and this has been done with all plots shown in figure 9. The multiplier is a function of coil length, and at a minimum there would need to be a new correction term as a function of coil length that would be used to multiply estimates generated from μ_{rod} . If this was done, it would look something like this if the correction function was symbolized by the capital greek letter gamma – $\Gamma(\cdot)$:

$$A_L = \mu_o \mu_{rod} \frac{A_f}{l_f} \Gamma \left(\frac{L_c}{L_f} \right)$$

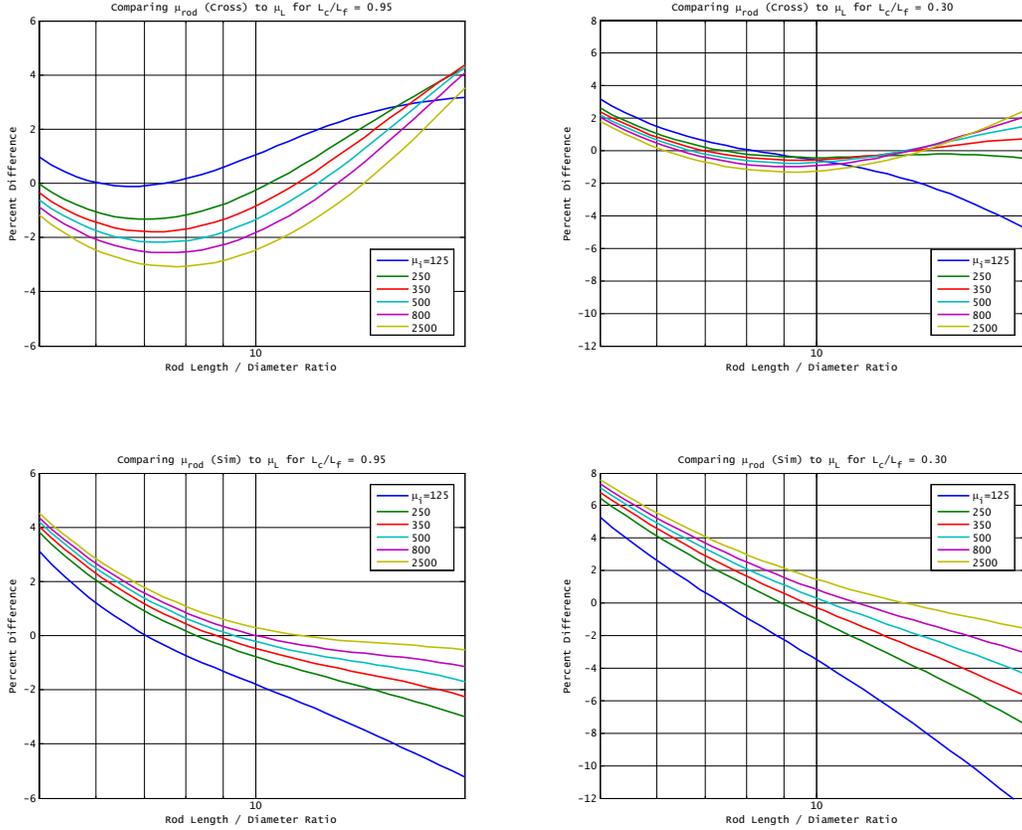


Figure 9: Difference between μ_{rod} and back-computed values

This leaves us with a quandry. On the one hand we could create a correction function for use with μ_{rod} values using Cross' formula for demagnetization factor. This would produce less accurate inductance estimates, but not that much worse (see the top two graphs of figure 9). With this option, there's still a complex formula for μ_{rod} in addition to either a polynomial or direct interpolation of simulation data for the correction factor, Γ . This might also create some confusion, as we would be recommending the use of a different estimate for μ_{rod} (based on simulation data) in calculating antenna effective height (see [osengr]).

Or, on the other hand, we could simply abandon μ_{rod} completely and define a new method for estimating inductance. This is the option that was chosen, because it eliminates some complexity and avoids confusion.

Example Software and Data

There is a zip file attached to the PDF file containing this article. It contains Matlab/Octave scripts and polynomial coefficient data which implement the methods described herein.

Also included are several sample ferrite material data files which can be used as a guide to generating files for actual ferrite permeability. These files were obtained from the Fair-Rite company's web site, and are included for example purposes only. We don't guarantee their contents, and they should not be used for design work without verification of the contained data. Please obtain permeability data for the ferrite to be used or verify the data files included are correct before using them.

Many PDF viewers are capable of extracting the attached zip file, as is the [PDFtk] utility (freeware). Check your PDF viewer's user manual or online help for more information.

What's It Worth?

A final comment about the accuracy of these estimates is in order. Simulations upon which the estimating functions are based use an exact value of ferrite permeability. With few exceptions, real world ferrite rods come with a large margin of error in specified permeability. A $\pm 20\%$ tolerance is not unusual.

Depending on the rod's aspect ratio, a wide tolerance on the permeability may or may not effectively limit the overall accuracy of inductance estimates. Inductance estimates for rods with a smaller aspect ratios and large values of μ_i will be less sensitive to variations in permeability.

This behavior is easily explored by examining inductance estimates using the formulas presented above, and the data tables attached to the PDF file. Two examples are shown in figure 10 where the effect of a $\pm 20\%$ variation in permeability is calculated for two different scenarios. In both cases, the rod diameter is 10mm, and coils are 70mm long, with 100 turns of wire.

- Nominal $\mu_i = 2000$ and rod length 80mm (rod aspect ratio 8:1)
- Nominal $\mu_i = 125$ and rod length 200mm (rod aspect ratio 20:1)

In the first case (smaller aspect ratio, high permeability), a $\pm 20\%$ swing in permeability changes the inductance by less than one percent. However, for a rod with low permeability and high aspect ratio, the same percentage swing in μ_i changes the inductance by a full $\pm 20\%$.

l_f/d_f	μ_i Variation	μ_i Value	L (μH)
8	-20%	1600	492.2
8	+20%	2400	496.0
20	-20%	100	590.6
20	+20%	150	721.7

Figure 10: Effect of μ_i tolerances on inductance

Feedback

We sometimes accept feedback on articles such as this. There's no guarantee we'll see or respond to reader inputs, but those who wish may visit the osengr.org web site and look for more information about feedback in the section about articles. For example, there may be a link to a WordPress blog where user inputs can be offered. This is all presented in a conditional form because the acceptance of, and method for reader inputs may change or vanish over time. Furthermore, we may not always monitor feedback blogs on a regular basis, so responses may be delayed significantly.

Inductive Permeability Table

ui = 63

L/D	Lc/Lf=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5.0000	2.5635	2.2094	1.9695	1.7802	1.6167	1.4735	1.3388	1.2107	1.0827	0.9405
7.0711	2.2664	1.9297	1.7065	1.5325	1.3858	1.2568	1.1387	1.0272	0.9167	0.7975
10.0000	1.9483	1.6302	1.4244	1.2671	1.1378	1.0252	0.9251	0.8318	0.7408	0.6438
14.1421	1.6234	1.3237	1.1363	0.9974	0.8862	0.7931	0.7119	0.6378	0.5674	0.4928
20.0000	1.3113	1.0325	0.8654	0.7462	0.6546	0.5803	0.5177	0.4626	0.4111	0.3570

ui = 125

L/D	Lc/Lf=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5.0000	2.8273	2.4634	2.2118	2.0097	1.8314	1.6733	1.5218	1.3759	1.2289	1.0639
7.0711	2.6026	2.2532	2.0149	1.8241	1.6586	1.5101	1.3707	1.2368	1.1023	0.9557
10.0000	2.3516	2.0176	1.7927	1.6141	1.4615	1.3246	1.1987	1.0782	0.9588	0.8306
14.1421	2.0684	1.7483	1.5372	1.3728	1.2345	1.1138	1.0041	0.9003	0.7994	0.6914
20.0000	1.7553	1.4513	1.2564	1.1087	0.9886	0.8859	0.7949	0.7111	0.6302	0.5445

ui = 250

L/D	Lc/Lf=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5.0000	2.9877	2.6182	2.3598	2.1501	1.9627	1.7956	1.6337	1.4769	1.3179	1.1388
7.0711	2.8262	2.4691	2.2215	2.0199	1.8421	1.6807	1.5269	1.3777	1.2269	1.0616
10.0000	2.6539	2.3099	2.0721	1.8784	1.7089	1.5537	1.4082	1.2667	1.1254	0.9729
14.1421	2.4542	2.1202	1.8914	1.7068	1.5461	1.4016	1.2667	1.1363	1.0075	0.8691
20.0000	2.2042	1.8818	1.6642	1.4911	1.3441	1.2130	1.0923	0.9778	0.8649	0.7446

ui = 500

L/D	Lc/Lf=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5.0000	3.0770	2.7045	2.4424	2.2284	2.0361	1.8638	1.6962	1.5332	1.3675	1.1803
7.0711	2.9579	2.5965	2.3435	2.1357	1.9507	1.7816	1.6193	1.4611	1.3005	1.1240
10.0000	2.8473	2.4974	2.2518	2.0488	1.8685	1.7017	1.5435	1.3884	1.2329	1.0645
14.1421	2.7311	2.3884	2.1479	1.9497	1.7731	1.6117	1.4585	1.3085	1.1593	0.9985
20.0000	2.5770	2.2421	2.0079	1.8152	1.6467	1.4922	1.3464	1.2056	1.0652	0.9152

ui = 1000

L/D	Lc/Lf=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5.0000	3.1243	2.7502	2.4861	2.2699	2.0749	1.9000	1.7292	1.5630	1.3938	1.2023
7.0711	3.0297	2.6661	2.4102	2.1990	2.0101	1.8369	1.6699	1.5067	1.3407	1.1581
10.0000	2.9582	2.6052	2.3553	2.1469	1.9606	1.7870	1.6216	1.4585	1.2948	1.1173
14.1421	2.9027	2.5550	2.3076	2.1010	1.9147	1.7428	1.5783	1.4160	1.2541	1.0791
20.0000	2.8349	2.4922	2.2473	2.0416	1.8584	1.6879	1.5246	1.3653	1.2056	1.0346

ui = 2000

L/D	Lc/Lf=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5.0000	3.1486	2.7737	2.5087	2.2913	2.0949	1.9187	1.7463	1.5783	1.4073	1.2136
7.0711	3.0674	2.7025	2.4452	2.2322	2.0412	1.8659	1.6964	1.5306	1.3618	1.1759
10.0000	3.0179	2.6633	2.4110	2.1998	2.0102	1.8331	1.6637	1.4964	1.3282	1.1457
14.1421	2.9993	2.6489	2.3976	2.1865	1.9947	1.8169	1.6460	1.4768	1.3076	1.1246
20.0000	2.9907	2.6435	2.3923	2.1789	1.9870	1.8068	1.6329	1.4624	1.2909	1.1071

ui = 4000

L/D	Lc/Lf=0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5.0000	3.1609	2.7856	2.5201	2.3022	2.1051	1.9281	1.7549	1.5861	1.4141	1.2194
7.0711	3.0866	2.7212	2.4631	2.2492	2.0572	1.8807	1.7100	1.5429	1.3726	1.1850
10.0000	3.0490	2.6935	2.4400	2.2273	2.0360	1.8570	1.6855	1.5160	1.3455	1.1604
14.1421	3.0507	2.6989	2.4456	2.2320	2.0374	1.8565	1.6821	1.5092	1.3362	1.1489
20.0000	3.0770	2.7275	2.4728	2.2551	2.0584	1.8729	1.6931	1.5164	1.3383	1.1474

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Revision History

1. Changed recommended interpolation method for μ_L and added more data to Matlab/Octave data file (attached document). Removed polynomial coefficients. Comparison to Bolton's data and local experiments updated to reflect new interpolation method.
2. Many text edits. Added:
 - “What’s it worth?” section
 - Inductive permeability table.
 - More information about μ_{rod} and reasoning for not using it.
 - Copy of inductance formula at beginning of article.

